Chapter 4
System of Independent Components

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(Version 0.1)

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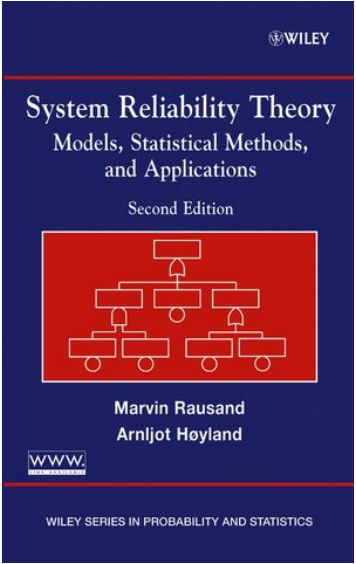
Slides related to the book

System Reliability Theory
Models, Statistical Methods,
and Applications

Wiley, 2004

Homepage of the book:
http://www.ntnu.edu/ross/books/srt



Independent components

Throughout this chapter we assume that all the n components of the system are independent.

Independence implies that the failure of one component will not have any influence on the other components - they will continue to operate as if nothing had happened.

The assumption of independence is not always realistic. We will study a different approach in Chapter 8.

State variables

We assume that the state variable of component i is a random variable $X_i(t)$ that is a function of time t.

The state vector of the system is

$$\mathbf{X}(t) = (X_1(t), X_2(t), \ldots, X_n(t))$$

The system state is given by

$$\phi(\mathbf{X}(t))$$

The n components are independent when $X_1(t), X_2(t), \ldots, X_n(t)$ are independent random variables.

Probabilities of interest

Reliabilities (= Function probabilities)

$$p_i(t) = \Pr(X_i(t) = 1) = \Pr(\text{Component } i \text{ is functioning at time } t)$$
$$p_S(t) = \Pr(\phi(\mathbf{X}(t) = 1) = \Pr(\text{The system is functioning at time } t)$$

Note that:

$$E[X_i(t)] = 0 \cdot \Pr(X_i(t) = 0) + 1 \cdot \Pr(X_i(t) = 1) = p_i(t)$$
$$E[\phi(\mathbf{X}(t)] = 0 \cdot \Pr(\phi(\mathbf{X}(t) = 0) + 1 \cdot \Pr(\phi(\mathbf{X}(t) = 1) = p_S(t)$$

When the components are independent, the system reliability $p_S(t)$ is a function of the $p_i(t)$'s alone, and we may write

$$p_S(t) = h(\mathbf{p}(t)) = h(p_1(t), p_2(t), \ldots, p_n(t))$$

Series structure - 1

The structure function of a series structure is

$$\phi(\mathbf{X}(t) = \prod_{i=1}^{n} X_i(t)$$

Since the components are independent, the system reliability is

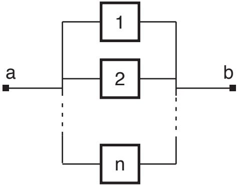
$$h(\mathbf{p}(t)) = E(\phi(\mathbf{X}(t)) = E\left(\prod_{i=1}^{n} X_i(t)\right) = \prod_{i=1}^{n} E(X_i(t)) = \prod_{i=1}^{n} p_i(t)$$

Series structure - 2

The reliability of a series structure of independent components is therefore obtained as the product of the component reliabilities.

$$p_S(t) = \prod_{i=1}^{n} p_i(t) \tag{1}$$

Parallel structure - 1



The structure function of a parallel structure is

$$\phi(\mathbf{X}(t)) \quad = \quad \coprod_{i=1}^{n} X_i(t) \quad = \quad 1 - \prod_{i=1}^{n}(1 - X_i(t))$$

Hence

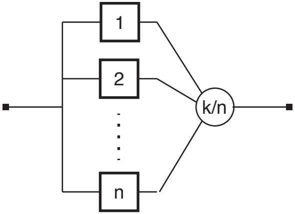
$$h(\mathbf{p}(t)) = E(\phi(\mathbf{X}(t))) = 1 - \prod_{i=1}^{n}(1 - E(X_i(t))) = 1 - \prod_{i=1}^{n}(1 - p_i(t))$$

Parallel structure - 2

The reliability of a parallel structure of independent components is therefore

$$p_S(t) = 1 - \prod_{i=1}^{n}(1 - p_i(t))$$

k-out-of-n structure - 1



The structure function of a k-out-of-n structure can be written as

$$\phi(\mathbf{X}(t)) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{n} X_i(t) \geq k \\ 0 & \text{if} \quad \sum_{i=1}^{n} X_i(t) < k \end{cases}$$

k-out-of-n structure - 2

A k-out-of-n structure is often called a *koon* structure.

Notice that:

- A 1oo1 structure is a single component
- An *noon* structure is a series structure of n components (All the n components must function for the structure to function)
- A 1oo$n$ structure is a parallel structure of n components (It is sufficient that one component is functioning for the structure to function)

k-out-of-n structure - 3

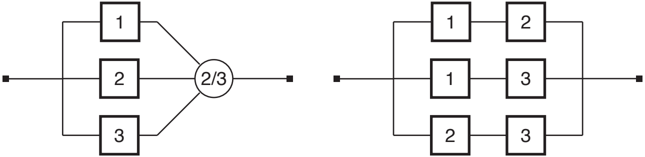
When all the n components have identical reliabilities $p_i(t) = p(t)$, the variable $Y(t) = \sum_{i=1}^{n} X_i(t)$ is binomially distributed $(n, p(t))$:

$$\Pr(Y(t) = y) = \binom{n}{y} p(t)^y (1 - p(t))^{n-y} \quad \text{for } y = 0, 1, \ldots, n$$

In this case

$$p_S(t) = \Pr(Y(t) \geq k) = \sum_{y=k}^{n} \binom{n}{y} p(t)^y (1 - p(t))^{n-y}$$

2-out-of-3 structure -1



The structure function of a 2-out-of-3 (2oo3) structure is

$$\begin{aligned}\phi(\mathbf{X}(t)) &= X_1X_2 \amalg X_1X_3 \amalg X_2X_3 \\ &= 1-(1-X_1X_2)(1-X_1X_3)(1-X_2X_3) \\ &= X_1X_2 + X_1X_3 + X_2X_3 - 2X_1X_2X_3\end{aligned}$$

Since the components are independent, the system reliability is

$$p_S(t) = p_1(t)p_2(t) + p_1(t)p_3(t) + p_2(t)_3(t) - 2p_1(t)p_2(t)p_3(t)$$

2-out-of-3 structure - 2

When the three components are identical, such that $p_i(t) = p(t)$ for $i = 1, 2, 3$, the system reliability becomes

$$\boxed{p_S(t) = 3p(t)^2 - 2p(t)^3}$$

Pivotal decomposition - 1

Pivotal decomposition is done with respect to a specified component i. Two situations are considered:

1. Component i is known to function, i.e., $X_i(t) = 1$. In this case, the structure function of the system may be written $\phi(1_i, \mathbf{X}(t))$
2. Component i is known to fail, i.e., $X_i(t) = 0$. In this case, the structure function of the system may be written $\phi(0_i, \mathbf{X}(t))$

where the symbol $\phi(1_i, \mathbf{X}(t))$ is introduced to pinpoint that component i is known to function, whereas the states of the other components are unknown – and similar for $\phi(0_i, \mathbf{X}(t))$.

Pivotal decomposition - 2

By pivotal decomposition, the structure function can now be written

$$\begin{aligned}\phi(\mathbf{X}(t)) &= X_i(t) \cdot \phi(1_i, \mathbf{X}(t)) + (1 - X_i(t)) \cdot \phi(0_i, \mathbf{X}(t)) \\ &= X_i(t) \cdot [\phi(1_i, \mathbf{X}(t)) - \phi(0_i, \mathbf{X}(t))] + \phi(0_i, \mathbf{X}(t))\end{aligned}$$

This formula is easy to prove. Because component i can only take the values 0 and 1, such that $X_i(t) = 0$ or $X_i(t) = 0$, we enter these two values into the above formula and get

1. When $X_i(t) = 1$, we get $\phi(\mathbf{X}(t)) = \phi(1_i, \mathbf{X}(t))$
2. When $X_i(t) = 0$, we get $\phi(\mathbf{X}(t)) = \phi(0_i, \mathbf{X}(t))$

Pivotal decomposition - 3

The system reliability can now be written

$$\begin{aligned} p_S(t) &= p_i(t) \cdot h(1_i, \mathbf{p}(t)) + (1 - p_i(t)) \cdot h(0_i, \mathbf{p}(t)) \\ &= p_i(t) \cdot [h(1_i, \mathbf{p}(t)) - h(0_i, \mathbf{p}(t))] - h(0_i, \mathbf{p}(t)) \end{aligned}$$

Notice that:

- $p_S(t)$ is a linear function of $p_i(t)$ when all the other component reliabilities are kept constant.
- The expression for $p_S(t)$ is nothing but what we obtain by using the law of total probability.
- Pivotal decomposition can be used to determine the reliability of any coherent structure.

Example: 2oo3 structure - 1

Consider a 2oo3 structure of independent and identical components. We perform pivotal decomposition with respect to component 3.

1. When component 3 is functioning, it is sufficient that 1-out-of-the remaining two components are functioning (i.e., the parallel structure of the two components is functioning)

$$h(1_3, \mathbf{p}(t)) = 2p(t) - p(t)^2$$

2. When component 3 is failed, both the two remaining components have to function (i.e., a series structure of the two components is functioning)

$$h(0_3, \mathbf{p}(t)) = p(t)^2$$

Example: 2oo3 structure – 2

The reliability of the 2oo3 structure is therefore by pivotal decomposition

$$p_S(t) = p_3(t)(2p(t) - p(t)^2) + (1 - p_3(t))p(t)^2$$

Since the components are equal, such that $p_3(t) = p(t)$, we obtain

$$\boxed{p_S(t) = 3p(t)^2 - 2p(t)^3}$$

Nonrepairable components

Now, assume that the components considered are not repaired, or that we are not interested in what happen to the components after they have failed. The reliability of component i is now

$$p_i(t) = \Pr(T_i > t) = R_i(t)$$

Series structure - 1

The survivor function of a series structure is

$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$

Since $R_i(t) = e^{-\int_0^t z_i(u)\,du}$, we have that

$$R_S(t) = \prod_{i=1}^{n} e^{-\int_0^t z_i(u)\,du} = e^{-\int_0^t \sum_{i=1}^{n} z_i(u)\,du}$$

The failure rate function $z_S(t)$ of a series structure (of independent components) is equal to the sum of the failure rate functions of the individual components:

$$z_S(t) = \sum_{i=1}^{n} z_i(t)$$

Series structure - 2

Example 4.5

Consider a series structure of n independent components with constant failure rates λ_i for $i = 1, 2, \ldots, n$. The survivor function is

$$R_S(t) = e^{-\left(\sum_{i=1}^{n} \lambda_i\right) t}$$

and the MTTF is

$$\text{MTTF} = \int_0^\infty e^{-\left(\sum_{i=1}^{n} \lambda_i\right) t}\, dt = \frac{1}{\sum_{i=1}^{n} \lambda_i}$$

When all the n components are equal with $\lambda_i = \lambda$ for all $i = 1, 2, \ldots, n$,

$$R_S(t) = e^{-n\lambda t} \quad \text{and} \quad \text{MTTF} = \frac{1}{n\lambda}$$

Parallel structure - 1

The survivor function of a nonrepairable parallel structure of independent components is

$$R_S(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

When all the components have constant failure rates $z_i(t) = \lambda_i$, for $i = 1, 2, \ldots, n$, then

$$R_S(t) = 1 - \prod_{i=1}^{n} (1 - e^{-\lambda_i t})$$

Notice that $R_S(t)$ can not be written as $e^{-\lambda_S t}$ and the parallel structure has therefore not a constant failure rate, even if all its components have constant failure rates.

Parallel structure - 2

Example 4.9

Consider a parallel structure of two independent components with failure rates λ_1 and λ_2 respectively. The survivor function is

$$R_S(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

The mean time to failure is

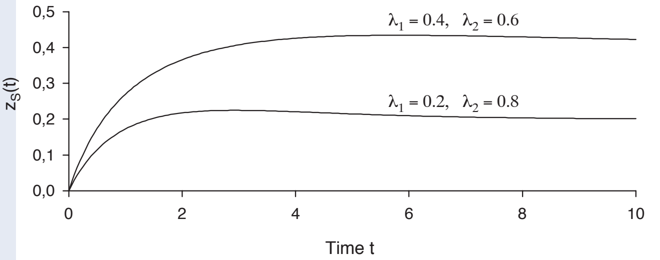
$$\text{MTTF} = \int_0^\infty R_S(t)\,dt = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2}$$

The failure rate function is

$$z_S(t) = -\frac{R_S'(t)}{R_S(t)} = \frac{\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - (\lambda_1 + \lambda_2)e^{-(\lambda_1+\lambda_2)t}}{e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}}$$

Parallel structure - 3

Example 4.9 (cont. 1)



The figure above is made for λ_1 and λ_2, such that $\lambda_1 + \lambda_2 = 1$. Notice that when $\lambda_1 \neq \lambda_2$, the failure rate function $z_S(t)$ will increase up to a maximum at a time t_0, and then decrease for $t \geq t_0$ down to $\min\{\lambda_1, \lambda_2\}$.

Parallel structure - 4

Example 4.9 (cont. 2)

For a parallel structure of two independent and identical components with failure rate λ, we have

$$R_S(t) = 2e^{-\lambda t} - e^{-2\lambda t} \quad \text{and} \quad f_S(t) = -R'_S(t) = 2\lambda \left(e^{-\lambda t} - e^{-2\lambda t}\right)$$

The failure rate function is

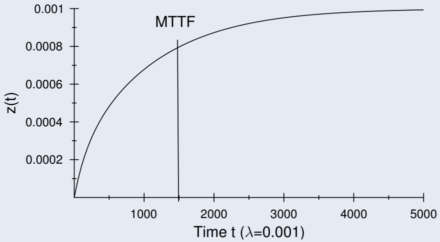
$$z_S(t) = \frac{f_S(t)}{R_S(t)} = \frac{2\lambda \left(e^{-\lambda t} - e^{-2\lambda t}\right)}{2e^{-\lambda t} - e^{-2\lambda t}} = \frac{2\lambda \left(1 - e^{-\lambda t}\right)}{2 - e^{-\lambda t}}$$

When $t \to \infty$, we observe that $z_S(t) \to \lambda$.

$z_S(t)\Delta t$ expresses the probability that a system that is functioning at time t will fail in $(t, t + \Delta t]$. After some time, it is likely that one of the two components has failed and that only one component, with failure rate λ, is performing the function.

Parallel structure - 5

Example 4.9 (cont. 3)



The failure rate function for a 1oo2 structure of independent and identical components with failure rate $\lambda = 0.001$ is shown in the figure above. The MTTF of the 1oo2 structure ($3/2\lambda = 1500$) is indicated in the figure. The failure rate is seen to approach λ when time t increases.

koon structure - 1

2oo3 structure

The structure function of a 2oo3 structure is

$$\phi(\mathbf{X}(t)) = X_1(t)X_2(t) + X_1(t)X_3(t) + X_2(t)X_3(t) - 2X_1(t)X_2(t)X_3(t)$$

The survivor function is

$$R_S(t) = R_1(t)R_2(t) + R_1(t)R_3(t) + R_2(t)R_3(t) - 2R_1(t)R_2(t)R_3(t)$$

koon structure - 2

2oo3 structure of identical components

When the three components are identical with the same constant failure rate λ, then

$$R_S(t) = 3\, e^{-2\lambda t} - 2e^{-3\lambda t}$$

The mean time to failure of the 2oo3 structure is

$$\text{MTTF} = \int_0^\infty R_S(t)\, dt = \frac{3}{2\lambda} - \frac{2}{3\lambda} = \frac{5}{6}\,\frac{1}{\lambda}$$

Notice that the MTTF of a 2oo3 structure of identical components is less than the MTTF of a single component of the same type.

koon structure - 3

2oo3 structure failure rate function

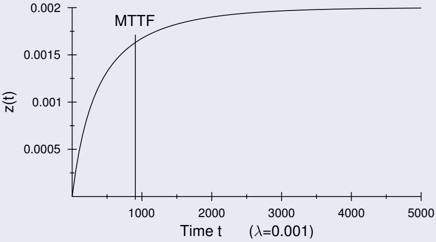
The failure rate function of a 2oo3 structure of independent and identical components with failure rate λ is

$$z_S(t) = \frac{-R_S'(t)}{R_S(t)} = \frac{6\lambda\left(e^{-2\lambda t} - e^{-3\lambda t}\right)}{3\,e^{-2\lambda t} - 2e^{-3\lambda t}} = \frac{6\lambda\left(1 - e^{-\lambda t}\right)}{3 - 2e^{-\lambda t}} \quad \xrightarrow[t\to\infty]{} 2\lambda$$

After some time, it is likely that one of the three components have failed such that the 2oo3 structure is functioning with only two components. If one of these fails (with rate 2λ), the structure fails. The failure rate function should therefore approach 2λ when t increases.

koon structure - 4

2oo3 structure failure rate function



The failure rate function for a 2oo3 structure of independent and identical components with failure rate $\lambda = 0.001$ is shown in the figure above. The MTTF of the 2oo3 structure $(5/6\lambda)$ is indicated in the figure. The failure rate is seen to approach 2λ when time t increases.

koon structure - 5

The survivor function is

$$R_S(t) = \sum_{x=k}^{n} \binom{n}{x} e^{-\lambda t x}(1 - e^{-\lambda t})^{n-x}$$

The mean time to failure is

$$\text{MTTF} = \sum_{x=k}^{n} \binom{n}{x} \int_0^{\infty} e^{-\lambda t x}(1 - e^{-\lambda t})^{n-x}\, dt$$

By introducing $v = e^{-\lambda t}$ we obtain (see Appendix A)

$$\text{MTTF} = \sum_{x=k}^{n} \binom{n}{x} \frac{1}{\lambda} \int_0^{1} v^{x-1}(1 - v)^{n-x}\, dv$$

$$= \frac{1}{\lambda} \sum_{x=k}^{n} \binom{n}{x} \frac{(x-1)!(n-x)!}{n!} = \frac{1}{\lambda} \sum_{x=k}^{n} \frac{1}{x}$$

MTTF of koon structures

$k\backslash n$	1	2	3	4	5
1	$\frac{1}{\lambda}$	$\frac{3}{2\lambda}$	$\frac{11}{6\lambda}$	$\frac{25}{12\lambda}$	$\frac{137}{60\lambda}$
2	–	$\frac{1}{2\lambda}$	$\frac{5}{6\lambda}$	$\frac{13}{12\lambda}$	$\frac{77}{60\lambda}$
3	–	–	$\frac{1}{3\lambda}$	$\frac{7}{12\lambda}$	$\frac{47}{60\lambda}$
4	–	–	–	$\frac{1}{4\lambda}$	$\frac{9}{20\lambda}$
5	–	–	–	–	$\frac{1}{5\lambda}$

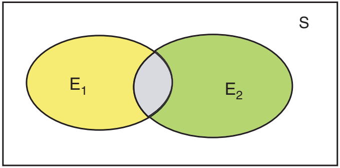
All the n components have the same failure rate λ

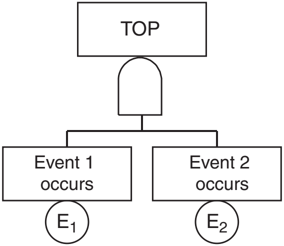
Fault tree notation

$$\begin{aligned} Q_0(t) &= \Pr(\text{The TOP event occurs at time } t) \\ q_i(t) &= \Pr(\text{Basic event } i \text{ occurs at time } t) \\ \check{Q}_j(t) &= \Pr(\text{Minimal cut set } j \text{ fails at time } t) \end{aligned}$$

- Let $E_i(t)$ denote that basic event i occurs at time t. $E_i(t)$ may, for example, be that component i is in a failed state at time t. Note that $E_i(t)$ does not mean that component i fails exactly at time t, but that component i is in a failed *state* at time t
- A minimal cut set is said to fail when all the basic events occur (are present) at the same time.

Single AND-gate





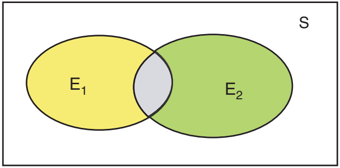
Let $E_i(t)$ denote that event E_i occurs at time t, and let $q_i(t) = \Pr(E_i(t))$ for $i = 1, 2$. When the basic events are independent, the TOP event probability $Q_0(t)$ is

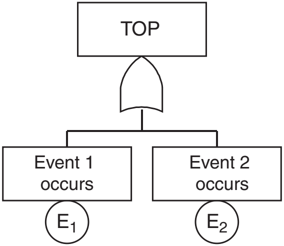
$$Q_0(t) = \Pr(E_1(t) \cap E_2(t)) = \Pr(E_1(t)) \cdot \Pr(E_2(t)) = q_1(t) \cdot q_2(t)$$

When we have a single AND-gate with m basic events, we get

$$Q_0(t) = 1 - \prod_{j=1}^{m} q_j(t)$$

Single OR-gate





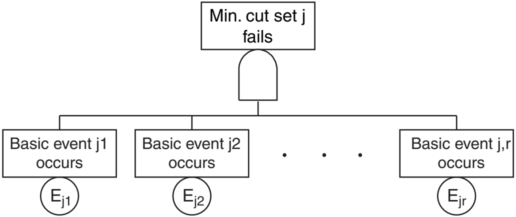
When the basic events are independent, the TOP event probability $Q_0(t)$ is

$$
\begin{aligned}
Q_0(t) &= \Pr(E_1(t) \cup E_2(t)) = \Pr(E_1(t)) + \Pr(E_2(t)) - \Pr(E_1(t) \cap E_2(t)) \\
&= q_1(t) + q_2(t) - q_1(t) \cdot q_2(t) = 1 - (1 - q_1(t))(1 - q_2(t))
\end{aligned}
$$

When we have a single OR-gate with m basic events, we get

$$
Q_0(t) = 1 - \prod_{j=1}^{m} (1 - q_j(t))
$$

Cut set assessment

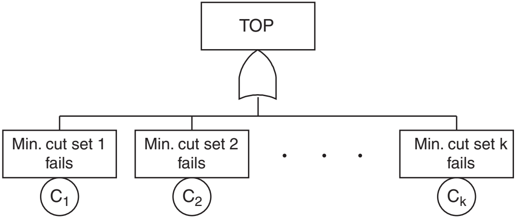


A minimal cut set fails if and only if all the basic events in the set fail at the same time. The probability that cut set j fails at time t is

$$\check{Q}_j(t) = \prod_{i=1}^{r} q_{ji}(t)$$

where we assume that all the r basic events in the minimal cut set j are independent.

TOP event probability



The TOP event occurs if at least one of the minimal cut sets fails. The TOP event probability is

$$Q_0(t) \leq 1 - \prod_{j=1}^{k} \left(1 - \check{Q}_j(t)\right) \tag{2}$$

The reason for the inequality sign is that the minimal cut sets are not always independent. The same basic event may be member of several cut sets. Formula (2) is called the *Upper Bound Approximation*.

What is redundancy?

☞ Redundancy: In an entity, the existence of more than one means for performing a required function [IEC 60050(191)]

☞ Redundancy: Existence of means, in addition to the means which would be sufficient for a functional unit to perform a required function or for data to represent information [IEC 61508, Part 4]

Redundant items may be classified as:

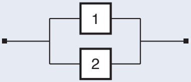
- Active (warm) redundancy
- Passive redundancy
  - Cold redundancy
  - Partly loaded redundancy

Active redundancy

Active redundancy means that two or more units are performing the same function at the same time. Active redundancy can be modeled by a standard parallel structure.

Active redundancy of order two

Two independent units with constant failure rates λ_1 and λ_2 are performing the same function – configured as a parallel structure.



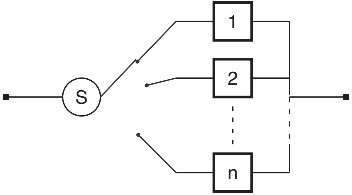
The survivor function of the system is

$$R_S(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t}$$

The survival probability is significantly increased compared to having only one unit.

Passive redundancy, perfect switching

Consider a system of n independent components with the same constant failure rate λ.



The survivor function of the system is

$$R_S(t) = \Pr\left(\sum_{i=1}^{n} T_i > t\right) = \sum_{k=0}^{n-1} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

That is, the Erlangian distribution with parameters n and λ

Cold standby, imperfect switching - 1

Consider a system of $n = 2$ components with failure rates λ_1 and λ_2. Let the probability of successful switching be $1 - p$.

The system may survive $(0, t]$ in two disjoint ways:

1. Item 1 does *not* fail in $(0, t]$ (i.e., $T_1 > t$).
2. Item 1 fails in a time interval $(\tau, \tau + d\tau]$ where $0 < \tau < t$. The switch S is able to activate item 2. Item 2 is activated at time τ and does not fail in the time interval $(\tau, t]$.

The probability of event 1 is

$$\Pr(T_1 > t) = e^{-\lambda_1 t}$$

Cold standby, imperfect switching - 2

The probability of event 2 is (for $\lambda_1 \neq \lambda_2$)

$$\begin{aligned} R_S(t) &= e^{-\lambda_1 t} + \int_0^t (1-p)\, e^{-\lambda_2(t-\tau)} \lambda_1 e^{-\lambda_1 \tau}\, d\tau \\ &= e^{-\lambda_1 t} + (1-p)\lambda_1 e^{-\lambda_2 t} \int_0^t e^{-(\lambda_1-\lambda_2)\tau}\, d\tau \\ &= e^{-\lambda_1 t} + \frac{(1-p)\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_2 t} - \frac{(1-p)\lambda_1}{\lambda_1-\lambda_2} e^{-\lambda_1 t} \end{aligned}$$

When $\lambda_1 = \lambda_2 = \lambda$ we get

$$\begin{aligned} R_S(t) &= e^{-\lambda t} + \int_0^t (1-p) e^{-\lambda(t-\tau)} \lambda e^{-\lambda \tau}\, d\tau \\ &= e^{-\lambda t} + (1-p)\lambda t e^{-\lambda t} \end{aligned}$$

Cold standby, imperfect switching - 3

The mean time to system failure is

$$\begin{aligned}\text{MTTF}_S &= \int_0^\infty R_S(t)\,dt = \frac{1}{\lambda_1} + \frac{(1-p)\lambda_1}{\lambda_1 - \lambda_2}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) \\ &= \frac{1}{\lambda_1} + (1-p)\frac{1}{\lambda_2}\end{aligned}$$

This result applies for all values of λ_1 and λ_2

Partly loaded standby, imperfect switching - 1

Consider a system of $n = 2$ components with failure rates λ_1 and λ_2 in active operation. Let λ_0 be the failure rate of component 2 in passive state, and let the probability of successful switching be $1 - p$.

The system may survive $(0, t]$ in two disjoint ways.

1. Item 1 does *not* fail in $(0, t]$ (i.e., $T_1 > t$).
2. Item 1 fails in a time interval $(\tau, \tau + d\tau)$, where $0 < \tau < t$. The switch S is able to activate item 2. Item 2 does not fail in $(0, \tau]$, is activated at time τ, and does not fail in $(\tau, t]$.

The probability of event 1 is

$$\Pr(T_1 > t) = e^{-\lambda_1 t}$$

Partly loaded standby, imperfect switching - 2

The probability of event 2 is (when $(\lambda_1 + \lambda_0 - \lambda_2) \neq 0$)

$$\begin{aligned} R_S(t) &= e^{-\lambda_1 t} + \int_0^t (1-p) e^{-\lambda_0 \tau} e^{-\lambda_2 (t-\tau)} \lambda_1 e^{-\lambda_1 \tau} \, d\tau \\ &= e^{-\lambda_1 t} + \frac{(1-p)\lambda_1}{\lambda_0 + \lambda_1 - \lambda_2} \left(e^{-\lambda_2 t} - e^{-(\lambda_0 + \lambda_1) t} \right) \end{aligned}$$

When $(\lambda_1 + \lambda_0 - \lambda_2) = 0$, the survivor function becomes

$$R_S(t) = e^{-\lambda_1 t} + (1-p)\lambda_1 t e^{-\lambda_2 t}$$

The mean time to system failure is

$$\text{MTTF}_S = \frac{1}{\lambda_1} + (1-p) \frac{\lambda_1}{\lambda_2(\lambda_1 + \lambda_0)}$$